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Operator with large spin and spinning D3-brane

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ABSTRACT: We consider the conformal dimension of an operator with large spin, using a spinning D3-brane with electric flux in $AdS_5 \times S^5$ instead of spinning fundamental string. This spinning D3-brane solution seems to correspond to an operator made by taking trace in a large symmetric representation. The conformal dimension, the spin and the R-charge show a scaling relation in a certain region of parameters. In the small string charge limit, the result is consistent with the fundamental string picture. There is a phase transition when the fundamental string charge become larger than a certain critical value; there is no stable D3-brane solution above the critical value.

KEYWORDS: D-branes, AdS-CFT Correspondence

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1 Introduction and summary

It is well known that in four-dimensional gauge theories the anomalous dimensions of composite operators with large Lorentz spin S scales logarithmically with the spin [1–4]. In [5], this logarithmic scaling at strong coupling in $\mathcal{N} = 4$ SYM was shown by using AdS/CFT correspondence [6]. They consider a folded string rotating in AdS_5 which is claimed to be dual to an operator of twist two with large spin in the gauge theory. The classical energy E of the string (scaling dimension of the operator) scales at large spin as

$$E = S + \frac{\sqrt{\lambda}}{\pi} \log S + \dots, \tag{1.1}$$

where λ is the 't Hooft coupling. This analysis is extended to the operator with large R -charge J as well as S , in the form of $\text{Tr}(D^S Z^J)$ (here $D = D_0 + D_1$ is the covariant derivative and Z is a complex scalar) which corresponds to a folded string rotating in both AdS_5 and S^5 in [7]. In the “long string limit,”

$$S \gg J \text{ and } \frac{\sqrt{\lambda}}{\pi J} \log \frac{S}{J} = \text{fixed}, \tag{1.2}$$

the classical energy of the string can be written in a closed form [4].

$$E = S + J \sqrt{1 + \frac{\lambda}{\pi^2 J^2} \log^2 \frac{S}{J}} + \dots \tag{1.3}$$

These operators belong to the $SL(2)$ sector in the gauge theory. Spectrum of such “long” operators can be analyzed using integrability of planar $\mathcal{N} = 4$ SYM [8]. Partial list of the recent development includes [9–35].

We want to consider what happens when this macroscopic string is replaced by a D-brane[36, 37]. There are several studies on this problem with Wilson lines in AdS/CFT [38, 39] which originally correspond to macroscopic fundamental strings. When the macroscopic string is replaced by a D-brane with electric flux[38, 40–43], it corresponds to a Wilson line of a higher representation; a D3-brane corresponds to the k -th symmetric representation while a D5-brane corresponds to the k -th anti-symmetric representation, where k is the string charge of the D-brane with electric flux.

Thus, replacing the folded rotating string by a folded rotating D3 or D5-brane, one could analyze the spectrum of the local operators in the k -th symmetric or anti-symmetric representation.¹ The spectrum of twist two operators in the k -th anti-symmetric representation is studied by Armoni [44] using rotating D5-brane. Another related study is “fat magnon”[45] which is a D3-brane version of the macroscopic string solution called “giant magnon”[46]. Other related solutions are found in [47, 48].

In this paper, we study the spectrum of the operator $\text{Tr}(D^S Z^J)$ in a symmetric representations using rotating D3-brane probe. We find classical D3-brane solutions rotating in both AdS_5 and S^5 . To use the holographic dictionary for Wilson lines and the more symmetries, we study the D3-brane counterpart of the “long string” (1.2). In the “long string” case, the folded string touches the boundary of AdS_5 (so it represents Wilson lines) and one more symmetry is enhanced (translation in χ . See the section 2). As a result, we find following scaling behavior in certain parameter regime.

$$(E - S)^2 - J^2 = T_3^2 f(\beta, \mu) \log^2 \frac{S}{J}, \tag{1.4}$$

$$\text{where, } \beta = \frac{J}{T_3 \log \frac{S}{J}}, \quad \mu = 2\pi \frac{\sqrt{\lambda} k}{N}, \quad T_3 = \frac{N}{2\pi^2}. \tag{1.5}$$

Which is valid when

$$\beta, \mu \text{ fixed,} \quad S \gg J, \quad N \rightarrow \infty, \quad \lambda \rightarrow \infty. \tag{1.6}$$

In small β and μ , the function f can be expanded as polynomial in β^2 and μ^2 .

$$f(\beta, \mu) = \mu^2 + c_{2,0}\beta^4 + c_{1,1}\beta^2\mu^2 + c_{0,2}\mu^4 + \text{higher order terms.} \tag{1.7}$$

Thus from (1.4), the anomalous dimension $\gamma := E - S - J$ can be written as

$$\gamma = J \sum_{m=0}^{\infty} \beta^{2m} \gamma_m(x^2), \tag{1.8}$$

$$= J \left[(\sqrt{1+x^2} - 1) + \beta^2 \left(\frac{c_{2,0} + c_{1,1}x^2 + c_{0,2}x^4}{2\sqrt{1+x^2}} \right) + o(\beta^4) \right], \tag{1.9}$$

$$\text{where, } x := \frac{\mu}{\beta} = \frac{k\sqrt{\lambda} \log(S/J)}{\pi J}. \tag{1.10}$$

¹More precisely Tr in the fundamental representation is replaced by the character (or Schur polynomial) of the symmetric or the anti-symmetric representation.

Note that the double expansion in β^2 and x^2 has same structure with the double expansion in $\frac{1}{N^2}$ and λ in the gauge theory side. At planar order (zeroth in β^2), the anomalous dimension coincide with that of k noninteracting folded strings (compare with (1.3)).

In some region in (β, μ) , there is no classical D3-brane solution. There exists a critical value for μ for each β above which the classical D3-brane solution does not exist (see figure 1). This seems to be a similar phenomenon as the phase transition in a symmetric Wilson loop observed in [41, 49, 50]. It would be interesting further work to check this phase transition in the gauge theory side.

In order to calculate this anomalous dimension in the gauge theory side, one should consider the limit $N \rightarrow \infty$ while keeping β, μ finite and λ small finite instead of the limit (1.6). In this limit certain kinds of non-planer diagrams also contribute to the result since μ kept finite. It will be an interesting future work to consider what kinds of diagrams contribute and what do not.

2 Set up

2.1 Symmetry, ansatz and boundary condition

First we will consider the symmetries of “infinity strings” in [5] which are dual to twist two operators with large spin, $S \gg \sqrt{\lambda}$. From those symmetries, we will find the appropriate ansatz and boundary conditions for the D3-brane which wraps 4 dimensional submanifold in AdS_5 and ends on the two light-like segments in the AdS_5 boundary. Then we will generalize the ansatz by turning on the angular momentum along S^5 .

The infinite string solution is given by [18]²

$$X_{-1}X_2 - X_0X_1 = 0, \quad X_3 = X_4 = 0. \quad (2.1)$$

Here $\{X_\mu\}$ are the Cartesian coordinates of $\mathbb{R}^{2,4}$ where the AdS_5 is embedded. In the global coordinates $\{\tilde{\tau}, \tilde{\rho}, \Omega_i\}$, ($i = 1, 2, 3, 4$) for AdS_5 ,

$$X_{-1} = \cosh \tilde{\rho} \cos \tilde{\tau}, \quad X_0 = \cosh \tilde{\rho} \sin \tilde{\tau}, \quad X_i = \sinh \tilde{\rho} \Omega_i, \quad \sum_{i=1}^4 \Omega_i^2 = 1, \quad (2.2)$$

the boundary is located at $\tilde{\rho} \rightarrow \infty$. The infinite string ends on the following two light-like lines at the boundary.

$$\tilde{\tau} = \tilde{\varphi} \quad \text{or} \quad \tilde{\tau} = \tilde{\varphi} + \pi, \quad \Omega_3^2 + \Omega_4^2 = 0. \quad \text{where } \tilde{\varphi} = \arctan \frac{\Omega_2}{\Omega_1}. \quad (2.3)$$

These two light-like Wilson lines preserve three symmetries of $SO(2, 4)$. These symmetries are more manifest in the $AdS_3 \times S^1$ foliation of AdS_5 .

$$\begin{aligned} (X_{-1}, X_0, X_1, X_2) &= \cosh \zeta (x_{-1}, x_0, x_1, x_2), & -x_{-1}^2 - x_0^2 + x_1^2 + x_2^2 &= -1, \\ (X_3, X_4) &= \sinh \zeta (x_3, x_4), & x_3^2 + x_4^2 &= 1, \\ ds^2(AdS_5) &= \cosh^2 \zeta ds^2(AdS_3) + \sinh^2 \zeta d\psi^2 + d\zeta^2. \end{aligned} \quad (2.4)$$

²Actually the folded string world-sheet covers (2.1) twice. Thus quantum numbers of the folded string should be doubled if one calculates them using (2.1). This two-foldedness should be taken into account in calculating quantum numbers for folded D3-brane.

We will use two coordinate systems for AdS_3 , $\{u, \chi, \sigma\}$ and $\{\tau, \rho, \varphi\}$. See appendix A. The infinite string (2.1) stretches along u, χ directions and located at $\zeta = 0, \sigma = 0$. And the three symmetries correspond to translations in u, χ and ψ [17]. Besides these continuous symmetries, there is an additional \mathbb{Z}_2 symmetry, $\sigma \leftrightarrow -\sigma$.

We will consider the D3-brane motion described by the DBI+WZ action

$$\begin{aligned}
 S_{D3} &= T_3 \int d^4 y L = T_3 \int d^4 y (L_{DBI} + L_{WZ}), \\
 L_{DBI} &= -\sqrt{-\det H}, & H_{\alpha\beta} &:= G_{MN}(Y) \frac{\partial Y^M}{\partial y^\alpha} \frac{\partial Y^N}{\partial y^\beta} + F_{\alpha\beta}, \\
 L_{WZ} &= -a C_{M_1 \dots M_4} \frac{\partial Y^{M_1}}{\partial y^{\alpha_1}} \dots \frac{\partial Y^{M_4}}{\partial y^{\alpha_4}} \epsilon^{\alpha_1 \dots \alpha_4} \frac{1}{4!},
 \end{aligned} \tag{2.5}$$

where Y^M , ($M = 0, \dots, 9$) denote the space-time coordinates and y^α , ($\alpha = 0, 1, 2, 3$) are the D3-brane world-volume coordinates. $F_{\alpha\beta}$ is the world-volume gauge flux. a is ± 1 depending on the choice of the orientation. The D3-brane tension T_3 is related to N by $T_3 = \frac{N}{2\pi^2}$ in our unit (AdS radius)=1.

We are going to find classical D3-brane solution which preserves the three symmetries and ends on the light-like segments (2.3) at the $\mathbb{R} \times S^3$ boundary. From three continuous symmetries, the ansatz for D3-brane is ($\{u, \chi, \psi, y\}$ are the world-volume coordinates)

$$F = b du d\chi, \quad \sigma = \sigma(y), \quad \zeta = \zeta(y). \tag{2.6}$$

To preserve the \mathbb{Z}_2 symmetry ($\sigma \leftrightarrow -\sigma$), we impose the following.

$$\frac{d\zeta}{d\sigma} = 0, \quad \text{when } \sigma = 0. \tag{2.7}$$

When $\zeta = 0$ where the size of S^1 shrinks, there may be conical singularity. To avoid this, we impose the following condition.

$$\frac{d\sigma}{d\zeta} = 0, \quad \text{when } \zeta = 0. \tag{2.8}$$

In the $AdS_3 \times S^1$ foliation, the $\mathbb{R} \times S^3$ boundary of AdS_5 is located at $\cosh^2 \zeta \cosh^2 \rho \rightarrow \infty$, or equivalently

$$\rho \rightarrow \infty \quad \text{or} \quad \zeta \rightarrow \infty. \tag{2.9}$$

And the two light-like lines at the boundary (2.3) becomes

$$\begin{aligned}
 \tau = \varphi \quad \text{or} \quad \tau = \varphi + \pi, \\
 \frac{X_3^2 + X_4^2}{X_{-1}^2 + X_0^2} = \frac{\sinh^2 \zeta}{\cosh^2 \zeta \cosh^2 \rho} \rightarrow 0.
 \end{aligned} \tag{2.10}$$

Under the ansatz (2.6), the D3-brane ends on the two segments (2.10) at the boundary (2.9) if and only if

$$\sigma(y), \zeta(y) = \text{finite}. \tag{2.11}$$

Equations (2.6), (2.7), (2.8), (2.11) are the summary of ansatz and conditions for D3-brane rotating in AdS_5 . These can be generalized by turning on the angular momentum along S^5 :

$$\theta = \nu u. \tag{2.12}$$

Here θ is the coordinate of a great circle of S^5 . Under these ansatz, the D3-brane action (2.5) becomes

$$\begin{aligned} S_{D3} &= T_3 \int dud\chi d\psi dy L, & L &= L_{DBI} + L_{WZ}, \\ L_{DBI} &= -\sqrt{\sinh^2 \zeta (\cosh^4 \zeta \cosh^2 2\sigma - b^2 - \nu^2 \cosh^2 \zeta) (\cosh^2 \zeta \sigma'^2 + \zeta'^2)}, \\ L_{WZ} &= -a(\cosh^4 \zeta - 1) \cosh 2\sigma \sigma'. \end{aligned} \tag{2.13}$$

Here a is ± 1 depending on the choice of the orientation. Under these ansatz, the equation of motion for the world-volume gauge field is automatically satisfied.

2.2 Quantum numbers

Here in this subsection we will obtain the expression of the conserved charges: the energy E , the spin S , the R-charge J , and the string charge k . First three charges E, S, J are calculated as the Noether charges from the spacetime isometry. Later we will calculate k by taking variation by NSNS B-field.

For a Killing vector ξ^M in $AdS_5 \times S^5$ and a small parameter ϵ , there is a symmetry of the action (2.5). Since this isometry also preserves the RR5-form field strength F_5 , the variation of 4-form potential should be written as

$$\delta C_4 = \epsilon d\Lambda_3, \tag{2.14}$$

where Λ_3 is a 3-form. The variation of the Lagrangian becomes

$$\delta L = \epsilon \partial_\alpha \left[-a \frac{1}{3!} \epsilon^{\alpha\alpha_2\alpha_3\alpha_4} \partial_{\alpha_2} Y^{M_2} \partial_{\alpha_3} Y^{M_3} \partial_{\alpha_4} Y^{M_4} \Lambda_{M_2 M_3 M_4} \right] =: \epsilon \partial_\alpha R^\alpha. \tag{2.15}$$

The Noether current j^α and the Noether charge Q for this symmetry is written as

$$j^\alpha = \frac{\partial L}{\partial(\partial_\alpha Y^M)} \xi^M - R^\alpha, \tag{2.16}$$

$$Q = T_3 \int d^3y j^0. \tag{2.17}$$

We only need to consider DBI-term in the action because we are considering folded D3-brane solution. Actually the terms in eq. (2.16) which come from the WZ-term cancel since two D3-branes have the opposite sign of the WZ-term to each other. The derivative of the DBI-term is given by

$$\frac{\partial L_{DBI}}{\partial(\partial_\alpha Y^M)} = -\sqrt{-\det H} (H_{\text{sym}}^{-1})^{\alpha\beta} G_{MN} \partial_\beta Y^N, \tag{2.18}$$

where H_{sym}^{-1} is the symmetric part of the inverse matrix of H .

We take u as the world-volume time. For the R-charge J the Killing vector is $\xi_J = \partial/\partial\theta$. The Noether charge is given as

$$J = T_3 \int d\chi \int d\psi \int dy j_J^u = 2\chi_0 T_3 \beta, \quad (2.19)$$

$$\beta := \int dy \frac{4\pi\nu(\cosh^2 \zeta \sigma'^2 + \zeta'^2) \sinh \zeta \cosh^2 \zeta}{\sqrt{(\cosh^4 \zeta \cosh^2 2\sigma - b^2 - \nu^2 \cosh^2 \zeta)(\cosh^2 \zeta \sigma'^2 + \zeta'^2)}}, \quad (2.20)$$

where χ_0 is the cut-off of the χ integral; χ is limited to $-\chi_0 \leq \chi \leq \chi_0$. As the same way, the Killing vector for $E - S$ is $\xi_{E-S} = -\partial/\partial\tau - \partial/\partial\varphi = -\partial/\partial u$ (see eqs. (A.11) and (A.12)), and the Noether charge is obtained as

$$E - S = 2\chi_0 T_3 \alpha, \quad (2.21)$$

$$\alpha := \int dy \frac{4\pi(\cosh^2 \zeta \sigma'^2 + \zeta'^2) \sinh \zeta \cosh^4 \zeta \cosh^2 2\sigma}{\sqrt{(\cosh^4 \zeta \cosh^2 2\sigma - b^2 - \nu^2 \cosh^2 \zeta)(\cosh^2 \zeta \sigma'^2 + \zeta'^2)}}. \quad (2.22)$$

On the other hand, for the spin S , the components of the Killing vector behave as $\xi_S \sim e^{2\chi}$ in large χ (see eq. (A.12)). Thus the charge S after integral over χ behaves as

$$S \sim T_3 e^{2\chi_0}, \text{ or } 2\chi_0 \sim \log \frac{S}{J}. \quad (2.23)$$

As a result we obtain the scaling behavior

$$(E - S)^2 - J^2 = T_3^2 (\alpha^2 - \beta^2) \log^2 \frac{S}{J}. \quad (2.24)$$

Let us turn to the string charge k . For a variation of B-field $\delta B_{u\chi}$, the variation of the action and the string charge k are related as ($\alpha' = \frac{1}{\sqrt{\lambda}}$ is the slope parameter in our unit.)

$$\delta S_{D3} = \frac{k}{2\pi\alpha'} \int dud\chi \delta B_{u\chi}. \quad (2.25)$$

Hence the string charge k is expressed as

$$k = 2\pi\alpha' T_3 \int d\psi \int dy \frac{\partial L}{\partial b} = \frac{N}{2\pi\sqrt{\lambda}} \mu, \quad (2.26)$$

$$\mu := \int dy \frac{4\pi b \sinh \zeta (\cosh^2 \zeta \sigma'^2 + \zeta'^2)}{\sqrt{(\cosh^4 \zeta \cosh^2 2\sigma - b^2 - \nu^2 \cosh^2 \zeta)(\cosh^2 \zeta \sigma'^2 + \zeta'^2)}}. \quad (2.27)$$

The scaling function $f(\beta, \mu)$ in eq. (1.4) is obtained from (2.24) by expressing $\alpha^2 - \beta^2$ in terms of β and μ .

3 Numerical analysis

So far we describe the general procedure for obtaining a D3-brane solution which is dual to the composite operator $\text{Tr}(D^S Z^J)$ in symmetric representations. In this section, we will find numerical solutions and analyze its phase structure and energy spectrum.

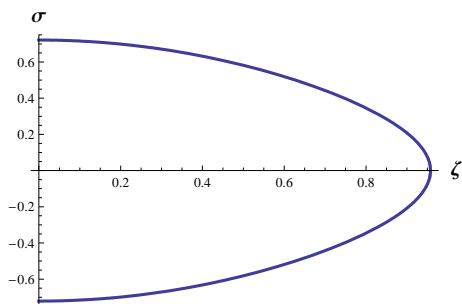


Figure 1. D3-brane example ($\nu = 0.999, b = 0.1$).

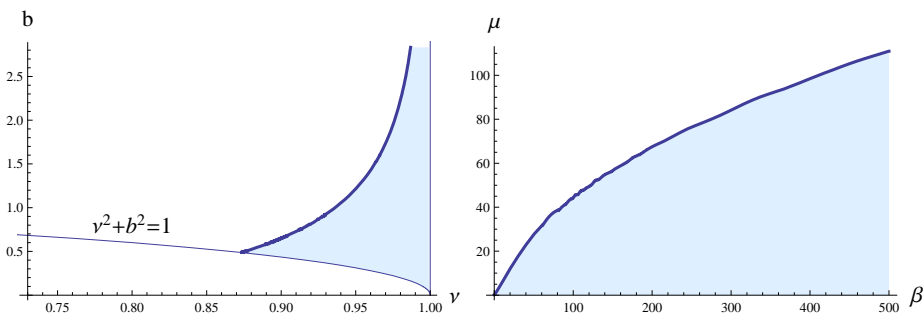


Figure 2. left: the region stable solutions exist, right: (β, μ) of the solutions fill the colored region.

3.1 Phase structure

The equation of motion derived from the action (2.13) is too complicated to solve it analytically. Thus, we find solutions numerically. The solution which satisfies the conditions (2.7), (2.8), (2.11) looks like an ellipse in (ζ, σ) -plane (see the figure 1). For some values of (ν, b) , there are several solutions. But if we impose stability condition,³ only one or no solution survives. And for some other values of (ν, b) , there's no solution (even unstable one). Figure 2 shows the region in (ν, b) where the stable solutions exist. The region is surrounded by following three curves.

- $\nu^2 + b^2 = 1$.

To avoid the Lagrangian (2.13) being an imaginary number, there's a lower bound for the size of solutions.

$$r := \sqrt{\zeta^2 + \sigma^2} \geq \frac{1}{2} \operatorname{arccosh}(\sqrt{\nu^2 + b^2}) \quad (3.1)$$

When $\nu^2 + b^2$ approaches to 1, the bound becomes smaller and stable solutions tend to shrink to the point $r = 0$. Accordingly, the physical quantities (β, μ) of the solution become smaller.

³We check the stability numerically. We consider several small fluctuations $\{\delta\sigma, \delta\zeta\}$ around a solution. If the solution maximize the Lagrangian $\int d\chi d\psi dy L$ under the fluctuations, then it is considered as a stable one.

- $\nu = 1$.

If $\nu \geq 1$, there's no solution except unstable one. Stable solutions (and its β, μ) in the colored region become infinity when $\nu \rightarrow 1$. This bound for the angular velocity in S^5 direction also exists for the folded string solution case [7].⁴

- The upper curve.

We cannot find analytic expression for this curve. Just below the curve there are two solutions (1 stable + 1 unstable). The two solutions get closer to each other when approaching the upper curve and disappear simultaneously above the curve. This curve is mapped to the upper curve in the (β, μ) plane via stable solutions. It suggests that there's some phase transition across the curve. This result requires further study to understand the phase transition in the gauge theory side.

3.2 Expansion in small β, μ

The energy spectrum (1.4), which is valid in the limit (1.6), is wholly determined if we find expression $f := \alpha^2 - \beta^2$ in terms of β, μ . Although we cannot find its full analytic expression, we suggest the form of series expansion and obtain the exact values of the coefficients at the lowest order. Higher order coefficients can be obtained numerically.

Consider the limit (ν, b) approaching to the curve $\nu^2 + b^2 = 1$. In the limit, as mentioned above, the stable solution (and its β, μ) become smaller. From the expression for α, β, μ in section 2 and the fact that ζ, σ is very small, one can see that

$$\frac{f}{\mu^2} = \frac{\alpha^2 - \beta^2}{\mu^2} \simeq \frac{1 - \nu^2}{b^2} \rightarrow 1 \tag{3.2}$$

in the limit. This result gives

$$(E - S)^2 - J^2 = k^2 \frac{\lambda}{\pi^2} \log^2 \left(\frac{S}{J} \right) \quad \text{when } \beta, \mu \rightarrow 0. \tag{3.3}$$

This is nothing but the spectrum of k noninteracting folded strings!(cf. (1.3)).

Assuming the $f(\beta, \mu)$ is analytic near the origin $(\beta, \mu) = (0, 0)$, we propose following expansion

$$f(\beta, \mu) = \sum_{m,n} c_{m,n} \beta^{2m} \mu^{2n}, \quad m, n \geq 0. \tag{3.4}$$

Here we use the fact that $f(\beta, \mu)$ is even function in both β and μ .⁵ And eq. (3.2) imply that

$$c_{0,0} = 0, \quad c_{1,0} = 0, \quad c_{0,1} = 1. \tag{3.5}$$

Numerically, we check the expansion (3.4) up to fourth power of β, μ and obtain the value of $c_{2,0}, c_{1,1}, c_{0,2}$.

$$c_{2,0} = 0.0084\dots, \quad c_{1,1} = 0.0074\dots, \quad c_{0,2} = -0.023\dots \tag{3.6}$$

For small (β, μ) , $f(\beta, \mu)$ is well approximated by this expansion as shown in figure 3.

⁴ We fix $\frac{\omega}{\kappa} = 1$ and $\frac{r}{\kappa}$ in [7] corresponds to ν in this paper.

⁵When $\nu \leftrightarrow -\nu$, e.o.m does not change and the stable solution remains same. So were α, μ . But β changes its sign (2.20). Similar argument hold for the $b \leftrightarrow -b$ case (in this case, (α, β) remains same but μ changes its sign.).

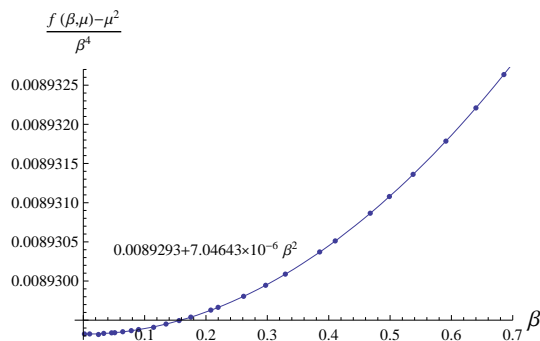


Figure 3. Along $\mu = 0.306857\beta$ we plot the graph. Its behavior agree with what we expected from (3.4).

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A Coordinates of AdS_3

We mainly use the coordinates of the $AdS_3 \times S^1$ foliation of AdS_5 (2.4). The coordinates of AdS_3 appeared in [17] is convenient for our purpose. We summarize its relation to the usual global coordinates. The coordinates (u, χ, σ) are given by

$$\begin{aligned}
 x_{-1} &= \cos u \cosh \sigma \cosh \chi - \sin u \sinh \sigma \sinh \chi, \\
 x_0 &= \sin u \cosh \sigma \cosh \chi + \cos u \sinh \sigma \sinh \chi, \\
 x_1 &= \cos u \cosh \sigma \sinh \chi - \sin u \sinh \sigma \cosh \chi, \\
 x_2 &= \cos u \sinh \sigma \cosh \chi + \sin u \cosh \sigma \sinh \chi.
 \end{aligned}
 \tag{A.1}$$

The metric in this coordinates is written as

$$ds^2(AdS_3) = -du^2 + d\chi^2 - 2 \sinh 2\sigma \, dud\chi + d\sigma^2.
 \tag{A.2}$$

On the other hand, the global coordinates (τ, ρ, φ) parametrize the AdS_3 as

$$\begin{aligned}
 x_{-1} &= \cosh \rho \cos \tau, & x_0 &= \cosh \rho \sin \tau, \\
 x_1 &= \sinh \rho \cos \varphi, & x_2 &= \sinh \rho \sin \varphi.
 \end{aligned}
 \tag{A.3}$$

The metric in this global coordinates is written as

$$ds^2(AdS_3) = -\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\varphi^2.
 \tag{A.4}$$

These two coordinate systems are related as

$$\sinh \rho = \sqrt{\cosh^2 \sigma \sinh^2 \chi + \sinh^2 \sigma \cosh^2 \chi}, \quad (\text{A.5})$$

$$\tan \tau = \frac{\tan u + \tanh \sigma \tanh \chi}{1 - \tan u \tanh \sigma \tanh \chi}, \quad (\text{A.6})$$

$$\tan \varphi = \frac{\tanh \sigma + \tan u \tanh \chi}{\tanh \chi - \tan u \tanh \sigma}, \quad (\text{A.7})$$

or

$$\sinh 2\sigma = \sinh 2\rho \sin(\varphi - \tau), \quad (\text{A.8})$$

$$\sinh 2\chi = \frac{\sinh 2\rho \cos(\varphi - \tau)}{\sqrt{1 + \sinh^2 2\rho \sin^2(\varphi - \tau)}}, \quad (\text{A.9})$$

$$e^{4iu} = e^{2i(\tau+\varphi)} \frac{\cos(\varphi - \tau) - i \cosh 2\rho \sin(\varphi - \tau)}{\cos(\varphi - \tau) + i \cosh 2\rho \sin(\varphi - \tau)}. \quad (\text{A.10})$$

The Killing vectors corresponding to the energy and the angular momentum are given in the (u, χ, σ) coordinates as

$$\partial_\tau = \frac{1}{2} \left(1 + \frac{\cosh 2\chi}{\cosh 2\sigma} \right) \partial_u + \frac{1}{2} \cosh 2\chi \tanh 2\sigma \partial_\chi - \frac{1}{2} \sinh 2\chi \partial_\sigma, \quad (\text{A.11})$$

$$\partial_\varphi = \frac{1}{2} \left(1 - \frac{\cosh 2\chi}{\cosh 2\sigma} \right) \partial_u - \frac{1}{2} \cosh 2\chi \tanh 2\sigma \partial_\chi + \frac{1}{2} \sinh 2\chi \partial_\sigma. \quad (\text{A.12})$$

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